Pre-Calculus CP 1 – Section 3.5 Notes **Name:** Key

EVEN MORE Applications of Exponential and Logarithmic Functions

Half Life

When examining half-life, the important thing to remember is that half-life represents the amount of time for half of the substance to disappear.

You will often use the Law of Uninhibited Growth and Decay when doing half-life problems:

$$A(t) = A_0 e^{kt} \qquad A = Pe^{rt}$$

Examples

1) Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$ where A_0 is the initial amount present and A is the amount present at time t (in days). What is the half-life of Iodine 131?

$1 = 2e^{-0.087t}$

$\frac{1}{2} = e^{-0.087t}$

$\ln\frac{1}{2} = -0.087t$

$t = \frac{\ln\frac{1}{2}}{-0.087} \approx 7.97$ days

2) The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

$A(t) = A_0 e^{kt}$

$1 = 2e^{1690k}$

$\ln\frac{1}{2} = 1690k$

$k = \frac{\ln\frac{1}{2}}{1690} \approx -0.00041$

$A(50) = 10e^{-0.00041(50)} \approx 9.8$ grms

Newton's Law of Cooling

The temperature, u, of a heated object at a given time, t, can be modeled by the following function:

$$u(t) = T + (u_0 - T)e^{kt},\ k < 0$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

Example

An object is heated to 100 degrees Celsius and is then allowed to cool in a room whose air temperature is 30 degrees Celsius.

u_0 T

a) If the temperature of the object is 80°C after 5 minutes, when will the temperature be 50°C?

t

$$80 = 30 + (100 - 30)e^{k(5)}$$
$$50 = 70e^{5k}$$
$$\frac{5}{7} = e^{5k}$$
$$\ln\frac{5}{7} = 5k$$
$$k = \frac{\ln\frac{5}{7}}{5} = -0.0673$$

$$50 = 30 + 70e^{-0.0673t}$$
$$\frac{2}{7} = e^{-0.0673t}$$
$$\ln\frac{2}{7} = -0.0673t$$
$$t = \frac{\ln\frac{2}{7}}{-0.0673} \approx 18.62 \text{ mins}$$

b) Determine the elapsed time before the temperature of the object is 35°C.

$$35 = 30 + 70e^{-0.0673t}$$
$$\frac{1}{14} = e^{-0.0673t}$$
$$t = \frac{\ln\frac{1}{14}}{-0.0673} \approx 39.22 \text{ mins}$$

c) What do you notice about $u(t)$, the temperature, as time passes?

It gets closer to room temp. (30°)

Logistic Growth Model

This is exponential growth, but with inhibitions…in other words, certain factors will *limit* the growth:

$$P(t)=\frac{c}{1+ae^{-bt}},\ b>0\ \&\ c>0$$

where a, b, and c are constants. c is called the *carrying capacity* because it is the ceiling for how big this value can become.

Example

Fruit flies are placed in a half-pint milk bottle with a banana and yeast plants. Suppose that the fruit fly population after t days is given by $P(t)=\frac{230}{1+56.5e^{-0.37t}}$.

a) What is the carrying capacity of the half-pint bottle?

230 flies

b) How many fruit flies were initially placed in the half-pint bottle?

$$P(0)=\frac{230}{1+56.5e^{-0.37(0)}}$$

$$=\frac{230}{1+56.5}=4 \text{ flies}$$

c) When will the population of fruit flies be 180?

$$180=\frac{230}{1+56.5e^{-0.37t}}$$

$$1+56.5e^{-0.37t}=\frac{23}{18}$$

$$e^{-0.37t}=\frac{5/18}{56.5}=.004916$$

$$\ln .004916=-0.37t$$

$$t=\frac{\ln .004916}{-0.37}\approx 14.37 \text{ days}$$